Information and Financial Markets

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Abstract. In this paper, we provide a model of an incomplete market economy with nominal assets and differential information. We prove an existence result of Walrasian expectation equilibria. Finally, we exhibit an example which shows that a differential information situation can remove the real indeterminacy of equilibria.

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1 Introduction

This paper provides a new model of financial markets with differential information by considering an economy where markets are incomplete and adding private information structures. That is, we incorporate differential information into a model of incomplete markets.

We consider an economy with a finite number of financial assets and a finite number of states of nature and we go a step further by introducing asymmetric or differential information into the model. We allow for information considerations within and incomplete markets setting as Radner (1968) introduced differential information into the Arrow-Debreu model. In particular, we assign to each agent, in addition to her initial endowment and utility function, a private information set, which is a partition of the exogenously given set which describes the states of nature. Then, private and differential information basically means a restriction on the consumption set of each agent.

Therefore, the model we consider involves financial assets and asymmetric information and then we extend a differential information approach to economies with incomplete financial markets. In the definition of equilibrium (called in this paper Walrasian expectation equilibrium), the information of an agent places a restriction on her admissible consumption bundles. This restriction can be formalized either as additional explicit constraints in the budget set or by considering preferences defined over the subset of bundles that an agent is able to perceive. Both approaches are equivalent and interpret private information of an agent as a restriction in her consumption set. Actually, better information allows for more contingent trades by enlarging the agents consumption set. Thus, in a Walrasian expectation equilibrium better informed agents, ceteris paribus, are never worse off than those with worse information, that is, an equilibrium rewards the information advantage of a trader.

We remark that the notion of equilibrium is essentially the same considered for an economy with financial assets. The only difference is that the consumption set for each agent depends on her private information about states of nature and then the budget sets are restricted due to differential information of agents. This illustrates the power of the Walrasian approach to general equilibrium: it allows us to deal with the dimension of time, space, uncertainty and differential information simply by reinterpreting the commodity space.

After stating the model, under standard assumptions, we obtain an existence result of Walrasian expectation equilibria. On the other hand, it is known that the indeterminacy of equilibria in incomplete financial markets is not only generically nominal, as in the case of complete markets, but has real implications since the equilibrium price indeterminacy is associated with indeterminacy in the commodity equilibrium allocations (see Balasko and Cass (1989) and Geanakoplos and Mas-Colel (1989)). The generic ocurrence of a continuum of equilibrium allocations, hurts the explicative and predictive role of general equilibrium theory. Several attempts tried to deepen the model structure and overcome this difficulty (see, for instance, Magill and Quinzii (1992), Pesendorfer (1995), Bissin (1998) and Faias, Moreno-García and Pascoa (2002)). In this paper, we adopt a different procedure and propose a differential information approach to markets with nominal assets. We study whether the presence of differences in information among agents allows for a determinacy of equilibrium. Indeed, we present an example which shows that a differential information structure within a nominal assets economy removes the indeterminacy of equilibria. However, more work is needed to obtain general results regarding private information structures and real indeterminacy of equilibria in an economy with nominal assets and differential information.

The remainder of the paper is organized as follows. Section 2 states the model of a differential information economy with nominal assets and discuss the assumptions. Section 4 contains some remarks on free disposal of commodities at equilibrium. Setcion 4 shows and existence result of Walrasian expectation equilibria and state conditions on both the return matrix and the information structure which guarantee the differentiability of the demand functions. Finally, Section 5 includes and example which points out how differential information can remove the real indeterminacy of equilibria.

2 A Model with Financial Assets and Differential Information

In this Section, we provide a model of an incomplete financial market economy $\mathcal{E}$ with differential information. For it, we consider a general equilibrium model with incomplete financial markets, which is essentially described in Werner (1985), Geanakoplos and Polimarchakis (1986) and in Geanakoplos and Mas-

Colell (1989), and we introduce differential information structures as in Radner (1968) (see also Yannelis (1991)).

The economy evolves over two periods ($t = 0, 1$) with S possible states of nature ($s = 1, \ldots, S$) in the second period. There are L physical commodities ($\ell = 1, \ldots, L$) available in $t = 0$. At date $t = 1$, in each state of the world, L spot markets open, where the L consumption commodities are traded.

Let $\mathcal{S}$ denote the finite set of possible states of nature, i.e., $\mathcal{S} = \{1, \ldots, S\}$. Let $\mathcal{P}$ denote the set of partitions of $\mathcal{S}$. An element $\mathbb{P} \in \mathcal{P}$ is called an *information set*. The interpretation is that states contained in an element $P \in \mathbb{P}$ cannot be distinguished under that information set. For each $s \in \mathcal{S}$ denote by $\mathbb{P}(s)$ the element of the partition $\mathbb{P}$ that contains s.

There are N consumers ($i = 1, \ldots, N$) in the economy. Each consumer i is characterized by a private information $\mathbb{P}_i$ which is a partition of $\mathcal{S}$, a preference relation on $\mathbb{R}_+^{L(S+1)} \times \mathbb{R}_+^{L(S+1)}$ represented by the utility function U_i and an initial endowment vector $\omega^i \in \mathbb{R}_+^{L(S+1)}$ with $\omega^i = (\omega_0^i, \omega_s^i, s = 1, \ldots, S)$ where ω_0^i denotes the initial endowment in $t = 0$ and ω_s^i denotes the initial endowment in $t = 1$ for the state s.

We will refer to a function with domain $\mathcal{S}$, constant on elements of $\mathbb{P}_i$, as $\mathbb{P}_i$-*measurable*, although, strictly speaking, measurability is with respect to the σ-algebra generated by the partition. We can think of such a function as delivering information to trader i, who can not discriminate between the states of nature belonging to any element of $\mathbb{P}_i$.

We state the following assumptions on endowments and preference relations for every consumer i:

(A.1) The utility function $U^i : \mathbb{R}_+^{L(S+1)} \to \mathbb{R}$ is continuous, increasing and quasi-concave.

(A.2) The initial endowment ω^i belongs to $\mathbb{R}_{++}^{L(S+1)}$ and is $\mathbb{P}_i$-measurable, that is, if $s' \in \mathbb{P}_i(s)$ then $\omega_s^i = \omega_{s'}^i$.

Assumption (A.2) states the compatibility of the initial endowment allocation with the private information structures in the sense that each agent is initially endowed with the same amount of commodities in states of nature that she is not able to distinguish.

Consumers can transfer wealth across states by trading in the financial markets in the first period. There is a finite number of nominal assets in the economy, indexed by $b = 1, \ldots, B.$. Each asset b promises to deliver $R^b(s) \in \mathbb{R}_+$ units of account, in each state of nature $s = 1, \ldots, S$ at the second period. Let $R = (R^b(s))_{b=1,\ldots,B}^{s=1,\ldots,S}$ denote the $S \times B$ payoff matrix describing the financial market structure of returns in this economy $\mathcal{E}$. We assume that assets are in zero supply, that is, agents trading these assets are actually trading promises to deliver the specified units of account in each state at date 1. In order to address an incomplete market framework, we assume that $B < S$. We also assume that R has full rank (i.e., the rank of R is B).

At date $t = 0$, agents take portfolio decisions and make consumption plans knowing that at t =1, when assets pay-off and consumption plans are carried out, different possible states of the world occurs taking into account their private information structures. This means that agents can trade the B assets and the L physical commodities at date 0 and can trade the L physical commodities at date 1 according to their information.

Therefore, the incomplete financial markets economy $\mathcal{E}$ with differential information is described by

$$\mathcal{E} \equiv (R, \mathbb{P}_i, U^i, \omega^i, i = 1, \ldots, N)$$

An allocation $x = (x^i, i = 1, \ldots, N)$ of commodities is *feasible* in the economy $\mathcal{E}$ if x is both *economically* and *informationally feasible*.

The allocation $x = (x^i, i = 1, \ldots, N)$ is *economically feasible* if $\sum_{i=1}^{N} x_0^i \leq \sum_{i=1}^{N} \omega_0^i$ and $\sum_{i=1}^{N} x_s^i \leq \sum_{i=1}^{N} \omega_s^i$ for every state $s \in \mathcal{S}$.

The allocation $x = (x^i, i = 1, \ldots, N)$ is *informationally feasible* if $(x_s^i, s \in \mathcal{S})$ is $\mathbb{P}_i$-measurable for every agent $i = 1, \ldots, N$.

A portfolio or asset allocation $y = (y^i, i = 1, \ldots, N) \in \mathbb{R}^{BN}$ is feasible if $\sum_{i=1}^{N} y^i = 0$.

Let $p_0 = (p_\ell)_{\ell=1}^{L}$ denote the price vector of commodities at time $t = 0$, let $p_s = (p_{s\ell})_{\ell=1}^{L}$ denote the spot price vector of commodities at time $t = 1$ in the state s. Let $p = (p_0, p_1, \ldots, p_S)$ denote the vector of commodity prices. The vector of asset prices at date 0 is denoted by $q \in \mathbb{R}^B$.

Given a price system $(p, q) \in \mathbb{R}_+^{L(S+1)} \times \mathbb{R}_+^B$, the budget set for agent i is given by

$$B^i(p,q) = \{(x,y) \mid \quad x \in \mathcal{X}_i,\ p_0 \cdot (x_0 - \omega_0^i) + q \cdot y \leq 0 \text{ and } \\ p_s \cdot (x_s - \omega_s^i) \leq R(s) \cdot y,\ \text{for all } s \in \mathcal{S}\}.$$

where $\mathcal{X}_i = \{x \in \mathbb{R}_+^{L(S+1)}$ such that $(x_s)_{s\in\mathcal{S}}$ is $\mathbb{P}_i$-measurable $\}$.

We remark that, given a price system, the possible consumption plans depend not only on endowments but also on the information structures which are different for different consumers. Hence, for each consumer i, the individual problem is:

$$\max_{(x,y)\in \mathbb{R}_+^{L(S+1)} \times \mathbb{R}^B} \quad U_i(x)$$

$$s.a. \qquad p_0 \cdot (x_0 - \omega_0^i) + q \cdot y \leq 0$$

$$p_s \cdot (x_s - \omega_s^i) \leq R(s) \cdot y, \quad \text{for each } s \in \mathcal{S}$$

$$x \in \mathcal{X}_i$$

As we have remarked in the introduction the private information of a agent i can be equivalently formalized by considering the utility function U_i restricted to $\mathcal{X}_i$.

Now, we can define de concept of equilibrium in our economy.

Definition 2.1 *A Walrasian expectation equilibrium for the nominal asset market economy $\mathcal{E} \equiv (R, \mathbb{P}_i, U^i, \omega^i, i = 1, \ldots, N)$ with differential information is a price system (p, q) and a feasible allocation of commodities and portfolios (x, y) such that*

(i) every agent i maximizes U^i on the budget constraint $B^i(p, q)$ and

(ii) $p_0 \cdot \sum_{i=1}^{N}(x_0^i - \omega_0^i) + \sum_{s=1}^{S} p_s \cdot \sum_{i=1}^{N}(x_s^i - \omega_s^i) = 0.$

Note that the consumers' problems are the same as those considered in the usual financial market economy models except the consumption sets that are restricted to be $\mathcal{X}_i, i = 1, ..., N$, instead of $\mathbb{R}_+^{L(S+1)}$. Condition (ii) is implied by the Walras law and ensures that in a Walrasian expectation equilibrium if a commodity is in excess supply its price is zero.

3 Equilibria and Free Disposal of Commodities

It is known that in an incomplete market setting we may have equilibria where some spot markets do not clear when preferences are not strictly monotone. In this Section, we show that in our model the equilibrium allocations can present free disposal of commodities due only to the restriction of consumption plans required by the private information of each consumer. Furthermore, we also state an assumption on the information structures which guarantees the market-clearing property at equilibrium.

For it, we state the following example of an economy, where the differential information is the source of free disposal at an equilibrium allocation.

Example 1. Consider the incomplete market economy with two agents 1 and 2, three states of nature $\{a, b, c\}$ and one commodity in each state. The endowments are $\omega^1 = (5, 5, 0)$ and $\omega^1 = (5, 0, 5)$. Both agents have the same preference relation which is represented by the utility function $U(x_a, x_b, x_c) = x_a^{1/2} + x_b^{1/2} + x_c^{1/2}$. The private information for agent 1 is given by $\mathbb{P}_1 = \{\{a, b\}, \{c\}\}$ and the private information for agent 2 is $\mathbb{P}_2 = \{\{a, c\}, \{b\}\}$.

In period $t = 0$ agents can transact two different assets with the following return matrix:

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

It can be shown that the the collection $q = (1/2, 1/2), p_a = 0, p_b = p_c = 1,\ y^1 = (-1, 1), y^2 = (1, -1), x^1 = (4, 4, 1), x^2 = (4, 1, 4)$ is a Walrasian expectation equilibrium with free disposal.

In order to obtain existence of equilibria with non negative prices and with no free disposal, we state the following assumption on the information structure:

(A.3) Given any state s there exists an agent $i \in N$ such that, $\{s\} \in \mathbb{P}_i$.

Note that, whenever there exists an agent who is completely informed the assumption above holds. Moreover, if the number of agents is much bigger than the set of states of nature, the hypothesis seems to be not very restrictive.

Proposition 3.1 *Consider a nominal asset market economy $\mathcal{E}$ under assumptions (A.1)-(A.3). Then any Walrasian expectation equilibrium is a non free* *disposal equilibrium. That is, if $(p,q),(x,y)$ is a Walrasian expectation equilibrium then $\sum_{i=1}^{N}(x_0^i-\omega_0^i)=0$ and $\sum_{i=1}^{N}(x_s^i-\omega_s^i)=0$, for every $s\in\mathcal{S}$. Therefore $p_0\gg 0$ and $p_s\gg 0$ for every state s.*

Proof. Let $(p,q),(x,y)$ be a Walrasian expectations equilibrium for the economy $\mathcal{E}$. By monotonicity of preferences $\sum_{i=1}^{N}(x_0^i-\omega_0^i)=0$ and $p_0\gg 0$. Suppose that $\sum_{i=1}^{N}x_s^{i,h}<\sum_{i=1}^{N}\omega_s^{i,h}$ for a state of nature s and for a physical commodity h. This implies $p_s=0$. By assumption (A.3), there exists an agent j who distinguishes s. Consider the consumption bundle z which coincides with x_j except for the commodity h and the state s, where $z_s^h=x_s^{j,h}+\left(\sum_{i=1}^{N}\omega_s^{i,h}-\sum_{i=1}^{N}x_s^{i,h}\right)$. Observe that z is $\mathbb{P}_j$-measurable and since $p_s^h=0$, we have $p_s\cdot z_s=p_s\cdot x_s$. Therefore, (z,y) belongs to $B_j(p,q)$ and by monotonicity of preferences, $U_j(z)>U_j(x_j)$, which is a contradiction.

Q.E.D.

4 Existence of Equilibrium

Note that the homogeneity of degree zero in prices of the first period budget constraint allows us to choose $(p_0,q)\in\Delta^{L+B-1}$ where

$$\Delta^{L+B-1}=\{(p_0,q)\in\mathbb{R}_+^{L+B}:\sum_{l=1}^{L}p_{0l}+\sum_{j=1}^{B}q_j=1\}.$$

The next Lemma shows the continuity property of the budget correspondences which will be used in the proof of our equilibrium existence result.

Lemma 4.1 *For every consumer i the budget correspondence B^i takes non empty and convex values and is continuous at every prices such that $(p_0,q)\in\Delta^{L+B-1}$ and $p_s\omega_s^i>0$, for every s.*

Proof. Since the allocation $(\omega,0)$ belongs to $B^i(p,q)$ for every (p,q), the correspondence has non-empty values and, by definition, $B^i(p,q)$ is convex for every (p,q).

Let (p^n, q^n) be a sequence of prices such that (p^n, q^n) converges to (p, q) and let (x^n, y^n) be a sequence such that $(x^n, y^n) \in B^i(p^n, q^n)$ and (x^n, y^n) converging to (x, y). Since x^{in} is $\mathbb{P}_i$-measurable for all n, we have that x^i is $\mathbb{P}_i$-measurable. Then, we conclude that the allocation (x, y) belongs to $B^i(p, q)$. Therefore, the correspondence B^i is upper-hemicontinuous.

Now, let p be a commodity price system such that $p_s \cdot \omega_s^i > 0$ and let us show that the interior of $B^i(p, q)$ is non empty. If $p_0 \cdot \omega_0^i > 0$, then $x = 0$ (which is obviously measurable for every information structure) together with a portfolio y such that $q \cdot y < p_0 \cdot \omega_0^i$ and $R(s) \cdot y > 0, s = 1, ..., S$ belongs to the interior of $B^i(p, q)$. Otherwise, $p_0 \cdot \omega_0^i = 0$. This implies that q is a non zero price vector and then we can take y such that $q \cdot y < 0$ and $p_s \cdot \omega_s^i + R(s) \cdot y > 0$ for every state s. Therefore, we can conclude that there is an interior point (x, y) in $B^i(p, q)$. Take a sequence (x^n, y^n) that converges to (x, y) and such that $x^n \in \mathcal{X}_i$ for every n. Then if (p^n, q^n) is a sequence of prices converging to (p, q), we have that for n large enough $p_0^n \cdot (x_0^n - \omega_0^i) + q^n \cdot y^n < 0$ and $p_s^n \cdot (x_s^n - \omega_s^i) < R(s) \cdot y^i$. Thus (x^n, y^n) belongs to the interior of $B^i(p^n, q^n)$ for n large enough, which implies that the correspondence given by the interior of the budget correspondencce is lower-hemicontinuous at (p, q). Since the closure of a lower-hemicontinuous correspondence is also lower-hemicontinuous the lower-hemicontinuity of B^i follows.

Q.E.D.

Let $x^i(p, q) \in \mathcal{X}_i$ denote the demand function of commodities for agent i at the price system (p, q). The asset price vector $q \in \mathbb{R}^B$ is a non-arbitrage price if and only if there is no $y \in \mathbb{R}^B$ with $q \cdot y = 0$ and $Ry > 0$. In order to obtain well-defined demand functions, we consider only those asset prices which satisfy the non-arbitrage condition.

Theorem 4.1 *Let $\mathcal{E}$ be a nominal asset market economy under assumptions (A.1)-(A.3). Then there exists a Walrasian expectation equilibrium $(p, q), (x, y)$ such that every price is strictly positive, $p_s \in \Delta^{L-1}$ for every $s \in \mathcal{S}$ and there is no free disposal.*

Proof. Given the economy $\mathcal{E}$ and a compact set $K \subset \mathbb{R}_+^{L(S+1)} \times \mathbb{R}^B$, with $(\omega, 0) \in K$, consider the generalized game played by the N consumers and by $S + 1$ auctioneers (one auctioneer for the first period and one auctioneer for each state of nature of the second period).

(i) Consumer i maximizes U_i on the budget set $B^i(p,q)\bigcap K$.

(ii) The first period auctioneer chooses prices $(p_0,q)\in\Delta^{L+B-1}$ that maximizes $p_0\cdot\sum_{i=1}^{N}(x_0^i-\omega_0^i)+q\cdot\sum_{i=1}^{N}y^i$.

(iii) The second period auctioneers choose prices $p_s\in\Delta^{L-1}$ that maximize $p_s\cdot\sum_{i=1}^{n}(x_s^i-w_s^i)$.

The strategy sets for the auctioneers are Δ^{L+B-1} and Δ^{L-1} respectively, which are non empty compact and convex. The payoff functions of the auctioneers are linear and then concave on their strategy variable and continuous on the strategy profile. By assumption (A.1), the payoff of consumers are quasi-concave and continuous and by Lemma 4.1, the constraint correspondences of the consumers have non-empty and convex values and are continuous. Moreover $B^i(p,q)\bigcap K$ is compact. Therefore, there exists an equilibrium for the generalized game.

Now, consider a sequence of increasing compact sets K^n. For each n, let $(\bar{x}^n,\bar{y}^n,\bar{p}^n,\bar{q}^n)$ be an equilibrium for the game restricted to K^n. Note that, $\sum_{i=1}^{N}\bar{x}^n\leq\sum_{i=1}^{N}\omega^i$, $(\bar{p}^n,\bar{q}^n)\in\Delta^{L+B-1}\times\Delta^{(L-1)S}$ and therefore there exists a converging subsequence with limit $(\tilde{x},\tilde{p},\tilde{q})$.

By Proposition 3.1 all prices are non null and there is no free disposal. Then, by inversion of a Cramer subsystem of budget equations, for each consumer, we can obtain in each truncated economy, $\bar{y}^{in}=\tilde{R}^{-1}[\bar{p}_s^n\cdot(\bar{x}_s^n-\omega_s^i)]_{s\in\beta}$, where $\tilde{R}$ is a non-singular submatrix of R, and β is the respective set of row indices. Now $\bar{y}^n$ converges to $\tilde{y}$ where $\tilde{y}^i=\tilde{R}^{-1}[\tilde{p}_s\cdot(\tilde{x}_s^n-\omega_s^i)]_{s\in\beta}$.

It remains to show that $(\tilde{x},\tilde{y},\tilde{p},\tilde{q})$ is an equilibrium for the economy $\mathcal{E}$. It is easy to check that $\tilde{x}^i$ is $\mathbb{P}_i$-measurable, $(\tilde{x}^i,\tilde{y}^i)\in B^i(\tilde{p},\tilde{q})$ for every agent i, and $(\tilde{x},\tilde{y})$ is physically feasible. Assume that $(\tilde{x}^i,\tilde{y}^i)$ is not an optimal choice for consumer i at prices $(\tilde{p},\tilde{q})$. Then, there exists $(\hat{x}^i,\hat{y}^i)$ in the interior of budget set such that $U^i(\hat{x}^i)>U^i(\tilde{x}^i)$. For n large enough and λ sufficiently close to zero we have $z=\lambda(\hat{x}^i,\hat{y}^i)+(1-\lambda)(\tilde{x}^i,\tilde{y}^i)$ belonging to $K^n\bigcap B^i(\bar{p}^n,\bar{q}^n)$. By convexity and continuity of preferences, $U^i(z)>U^i(\bar{x}^{in})$ for n large enough. This is a contradiction with the fact that $(\bar{x}^n,\bar{y}^n,\bar{p}^n,\bar{q}^n)$ is an equilibrium for all n.

Q.E.D.

The proof of the next result is delegated to an Appendix and it shows the differentiability properties of the demand functions. For it, we assume that R is in general position which means that every submatrix of R has full rank.

Proposition 4.1 *Let R be in general position. Assume that agent i distinguishes at least $S-B$ states and the utility function U^i is twice continuously differentiable, strictly increasing and concave. Then, the individual demand is a C^1 function.*

(See proof in the Appendix).

5 Information and Real Indeterminacy of Equilibria

It is known that the indeterminacy of equilibria in incomplete financial markets is not only generically nominal, as in the case of complete markets, but has real implications. That is, the equilibrium price indeterminacy is associated with indeterminacy in the commodity equilibrium allocations (see Balasko-Cass (1989) and Geanakoplos-Mas-Colell (1989)).

Example 2. Consider an economy with two agents, two states of nature, one commodity in each state and one asset with returns $R(1) = R(2) = 1$. As in the example stated by Bisin (1998) both agents have the same quasi-linear preference relation represented by the utility function $U(x) = x_0 - 1/2\sum_{s=1}^{2}(H - x_s)^2$. The initial endowments are $\omega^1 = (2H, 1, 1)$ for agent 1 and $\omega^2 = (4, 2, 4)$ for agent 2. It can be checked that an equilibrium is given by $p_0 = p_1 = p_2 = 1,\ q = 2H - 4,$ $y^1 = 1,\ x^1 = (4, 2, 2), x^2 = (2H, 1, 3)$. We remark that, given a positive real number λ, if we take the return matrix $R_\lambda(1) = \lambda,\ R_\lambda(2) = 1$ we obtain a different equilibrium allocation $(x^1_\lambda, x^2_\lambda)$ which is also an equilibrium allocation for the initial economy. Then, we have a real indeterminacy of equilibria and, as it is known the degree of indeterminacy is one. We remark that in the equilibria associated with $\lambda \neq 1$ the consumption for agent one differs between states. Consider now that agent 1 does not distinguishes between states 1 and 2 whereas agent 2 is fully informed. In this case, there are just two equilibrium allocations which are given by $x^1 = (4, 2, 2), x^2 = (2H, 1, 3)$ (corresponding to $\lambda = 1$) and the initial endowment allocation (corresponding to $\lambda \neq 1$), respectively.

We remark that, in the previous economy, when agent 1 does not distinguishes between states 1 and 2 different rates of inflation across the two states do not entail real effects on consumption. Moreover, this example is consistent with the intuition that differential information reduces the possibilities of trade. In fact, when the rate of inflation between state 1 and state 2 are not equal to 1, there is no transaction of the asset.

An important further study would be to obtain general results regarding private information structures and real indeterminacy of equilibria in an economy with nominal assets and differential information.

Appendix

Proof of Proposition 4.1.

Consider as numeraire the commodity one in each state. Given a partition $\mathbb{P}_i$ for consumer i, let $\widetilde{\mathbb{P}}_i = \{P_1^i, ..., P_{K^i}^i\}$ be the corresponding set of elements of the partition such that $card(P_k^i) > 1$, for all $k = 1, ..., K^i$. For each k, fix a state of nature $\bar{s}_k$ in each element $P_k^i \in \widetilde{\mathbb{P}}_i$. Therefore, the information structure of the consumer i, leads to $\rho = \sum_{k=1}^{K_i}(card(P_k^i) - 1)$ restrictions over the consumption set which are given by the equations, $x_{\bar{s}_k} - x_s = 0$, for all $s \in P_k^i$ such that $s \neq \bar{s}_k$, and for all $k = 1, ..., K^i$. For each $s \in P_k^i$, let γ_{ks} be the lagrange multiplier associated with the restriction $x_s - x_{\bar{s}_k} = 0$. Let μ be the lagrange multiplier associated with the budget constraint of the period 0 and $\lambda = (\lambda_1, ..., \lambda_S)$ denotes the lagrange multipliers vector for the budgets constraints of the period 1.

The lagrangian function for consumer i is given by:

$$\begin{aligned}\mathcal{L}^i(x_0, x_1, \lambda, y, \mu, \gamma) \quad = \quad & U^i(x_0, x_1) + \sum_{s=1}^{S} \lambda_s[R(s)y^i - p_s \cdot (x_s - w_s^i)] - \\ & \mu[q \cdot y + p_0 \cdot (x_0 - w_0^i)] + \sum_{k=1}^{K^i} \sum_{s \in P_k^i, s \neq \bar{s}_k} \gamma_{ks}(x_{\bar{s}_k} - x_s).\end{aligned}$$

Note that the last term exhibit the information structure of agent i. Indeed, if an agent is fully informed this term is removed and in this case the differentiability property of the demand was proved by Geanakoplos and Polemarchakis (1986).

The conditions which characterize the solution for the consumer problem are,

$$\begin{aligned}& D_0U^i - \mu p_0 = 0, \\ & D_sU^i - \lambda_s p_s + \Gamma_s = 0, s = 1, ..., S, \\ & R(s)y - p_s \cdot (x_s - w_s^i) = 0, s = 1, ..., S, \\ & -\mu q + \lambda' R = 0, \\ & -p_0 \cdot (x_0 - w_0^i) - q \cdot y = 0, \\ & x_{\bar{s}_k} - x_s = 0, \text{ for all } s \in P_k^i \text{ such that } s \neq \bar{s}_k, \text{ and for all } k = 1, ..., K^i,\end{aligned}$$

where Γ_s is a vector with L coordinates such that for all $l = 1, ..., L$,

$$(\Gamma_s)_l = \begin{cases} 0 & \text{if } s \notin \bigcup\limits_{k=1,...,K^i} P_k^i, \\ \sum\limits_{s' \in P_k^i, s' \neq \bar{s}_k} \gamma_{ks'} & \text{if } s \in P_k^i \text{ for some } k \in \{1,...,K_i\} \text{ and } s = \bar{s}_k, \\ -\gamma_{ks} & \text{if } s \in P_k^i \text{ for some } k \in \{1,...,K_i\} \text{ and } s \neq \bar{s}_k. \end{cases}$$

Let $\mathbf{J}$ be Jacobian matrix of order $S(L+1)+S+B+1+\rho$ given by the second order derivatives(with respect to $(x_0, x_1, \lambda, y, \mu, \gamma)$:

$$\mathbf{J} = \begin{bmatrix} D_0^2U^i & 0 & 0 & 0 & -p_0 & 0 \\ 0 & D_1^2U^i & -p_1 & 0 & 0 & V \\ 0 & -p_1' & 0 & R & 0 & 0 \\ 0 & 0 & R' & 0 & -q & 0 \\ -p_0 & 0 & 0 & -q' & 0 & 0 \\ 0 & V' & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $V = \begin{bmatrix} V_1 \\ \vdots \\ V_S \end{bmatrix}$ is the matrix of order $SL \times \rho$ defined as follows:

$$V_{sl,ks'} = \Big[\frac{\partial(D_sU^i - \lambda_s p_s + \Gamma_s)_l}{\partial\gamma_{ks'}}\Big] = \Big[\frac{\partial(x_{\bar{s}_k l} - x_{sl})}{\partial\gamma_{ks'}}\Big] =$$

$$\begin{cases} 0 & \text{if } s \notin \bigcup\limits_{k=1,...,K^i} P_k^i, \\ 1 & \text{if } s \in P_k^i \text{ for some } k \in \{1,...,K_i\} \text{ and } s = \bar{s}_k, \\ 0 & \text{if } s \in P_k^i \text{ for some } k \in \{1,...,K_i\},\ s \neq \bar{s}_k \text{ and } s \neq s', \\ -1 & \text{if } s \in P_k^i \text{ for some } k \in \{1,...,K_i\},\ s \neq \bar{s}_k \text{ and } s = s'. \end{cases}$$

It remains to prove that the matrix $\mathbf{J}$ is non-singular which, by applying the implicit function theorem, implies the continuous differentiability of demand function. Let us show that if $\mathbf{J}z = 0$ then $z = 0$. For it, assume that $\mathbf{J}z = 0$, with $z = (\hat{x}_0, \hat{x}_1, \hat{\lambda}, \hat{y}, \hat{\mu}, \hat{\gamma})$ that is,

$$\begin{aligned} &D_0^2U^i\hat{x}_0 - \hat{\mu}p_0 = 0 \\ &D_1^2U^i\hat{x}_1 - p_1\hat{\lambda} + V\hat{\gamma} = 0 \\ &-p_1'\hat{x}_1 + R\hat{y} = 0 \\ &R'\hat{\lambda} - \hat{\mu}q = 0 \\ &-p_0\hat{x}_0 - q'\hat{y} = 0 \\ &V'\hat{x}_1 = 0. \end{aligned}$$

Then $z'\mathbf{J}z = 0$, using $\mathbf{J}z = 0$, reduces to $\hat{x}_0'(D_0^2U^i)\hat{x}_0 + \hat{x}_1'(D_1^2U^i)\hat{x}_1 = 0$. The negative definiteness of D^2U^i implies $\hat{x}_0 = 0$ and $\hat{x}_1 = 0$. Since $p_0 \neq 0$ the first equality guarantees $\hat{\mu} = 0$. By using the third equality and noticing that R has full colinear rank, we obtain $\hat{y} = 0$. Moreover, if s is one of the $S - B$ states that the agent i distinguishes it follows that $\hat{\lambda}_s = 0$, by using the second equality with $p_s \neq 0$ and noticing that $V_s = 0$ for a state that the agent distinguishes. Next, we use the equation $R'\hat{\lambda} - \hat{\mu}q = 0$ with $\hat{\mu} = 0$ and the fact that R is in general position to obtain $\hat{\lambda}_s = 0$ for the remaining states s, that is, for the states that the agent does not distinguish. Finally, the second equality with $\hat{x}_1 = 0$ and $\hat{\lambda} = 0$ allows us to conclude that $\hat{\gamma} = 0$.

Q.E.D.

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