

A strategic model of club formation; existence and characterization of equilibrium

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Abstract. We introduce a new model of a club economy as a two stage game. Players derive utility from consumption of private good, consumption of public good, and the profile of crowding characteristics – those characteristics of a player that directly affect other players – of members of the same club. In the first stage of the game, players choose amounts to consume of an endowment of private good. The crowding characteristics acquired by a player are determined by his choice of consumption level, as is the amount of private good remaining to contribute to the production of the club good in the second stage of the game. In the second stage of the game, given the profile of crowding characteristics of the total player set, club memberships are endogenously determined as outcomes of subgame perfect equilibrium. We establish conditions for the existence of equilibrium and provide some examples illustrating that characterization results from models of club economies with price-taking equilibrium do not necessarily hold.

1 Introduction

The essential idea underlying models of economies with clubs (or Tiebout economies) is that the benefits of forming large clubs or jurisdictions are eventually offset or almost offset by negative externalities due to congestion or other problems associated with the organization of large groups of players. Congestion, however, may depend on more than just the numbers of individuals in a club; it may also depend on the crowding characteristics of those individuals that affect other members of the same club – whether other swimmers swim laps or play and splash about in a pool, are male or female, are macro-economists or econometricians, or smokers or nonsmokers. As these examples illustrate, some crowding characteristics of an individual – age for example – are inherent to the individual while others are acquired through consumption and life style choices – education and profession or whether to be a smoker, for example. There are now a number of models embodying these features and studying core and price taking equilibrium (see Conley and Smith 2005 and Kovalenkov and Wooders 2005 for recent, complimentary, surveys). While a number of papers in the literature also address questions of strategic club formation (cf., Demange 1994, 2005, Konishi, Le Breton and Weber 1997,1998, Conley and Konishi 2002), there are few papers in the literature taking a noncooperative approach to group formation in environments where individuals may have exogenously given or acquired crowding characteristics. To investigate noncooperative foundations (other than price-taking behavior) of cooperative outcomes in such economies is a natural direction of research. Since crowding characteristics may be acquired over time – for example, education, it is also natural to model club formation as a two-stage game where, in the first stage, individuals acquire crowding characteristics and, in the second stage, join clubs.

Our separation of the crowding effects of a player from other characteristics, such as tastes, which are presumed to have no direct effects on other members of the same club, was motivated by Conley and Wooders (1996,1997). In the context of these two papers and others in the more recent literature, restricting preferences and/or production possibilities to depend on crowding types enables decentralization of the core with anonymous prices. In our model, we assume that players acquire crowding characteristics through consumption of a private good. Moreover, choice of crowding type is a strategic variable and also has the realistic feature that the effects of a player on other players depends on the consumption choices of players.

Following Conley and Wooders (2001) we also consider situations where players may have different inherent abilities so that the same pattern of consumption by two different players does not necessarily lead to the two players acquiring the same crowding characteristics.¹ For each player we take as given a mapping, which depends on the unobservable inherent type of the player, from private consumption into crowding types. For example, a player may choose to acquire an education so that he can obtain a well paid job in a firm or he may choose to save his money to invest in a firm. The amount he must spend on each of these possibilities depends on the inherent characteristics of the player, his intelligence, for example. Alternatively, a player may spend income on acquiring business skills or retain income to go into a business partnership. Since some of our results apply to matching models, another example may be :A female player may choose to invest in education, anticipating joining a firm, or in her health, hoping to marry well and have many children, while a male may choose to invest in body building, hoping to attract a wealthy mate, or in income-earning abilities, hoping to attract a healthy mate and to have many children.

We assume that players behave strategically and design a Nash game with two stages. In the first stage of the game, players choose private consumption levels and consequently their crowding types. We assume private consumption bundles are available only in indivisible units. In the second stage, with the crowding profile of the economy already fixed, the players choose a mixed strategy which consists of a probability measure over the set of club locations. We note that our use of mixed strategies (lotteries) was motivated by Garratt and Qin (1996). In the context of choice of crowding type, a two-stage game is more natural than the static frameworks of Conley and Wooders, especially when crowding types are endogenous. For many crowding types, it is easy to imagine that choice or formation of crowding characteristics occurs before individuals join jurisdictions, clubs or firms. Education is a prime example.

¹Conley and Wooders (2001) call these inherent abilities ‘genetic types.’

2 The Model

2.1 Formal Elements

We consider an economy with one private good and I players indexed by $i \in \{1, \dots, I\} \equiv \mathcal{I}$. We denote by \mathcal{B} a set of possible private good consumption bundles, assumed to be finite; thus, we assume that commodities are indivisible.

There are T possible taste types of players, indexed by $t \in \{1, \dots, T\} \equiv \mathcal{T}$. A mapping $\tau : \mathcal{I} \rightarrow \mathcal{T}$ ascribes a taste type to each player in the economy.

There is a set of *crowding characteristics*, $\mathcal{C} = \{1, \dots, C\}$ and we assume that, as a by product of private consumption, a player acquires a *crowding type*, which is a probability measure over the set of crowding characteristics. The effect of private consumption on crowding type depends on characteristics inherent to the player, which we will call the player's *inherent type*. Accordingly, we assume that each player is also endowed with a inherent type, $g \in \{1, \dots, G\} = \mathcal{G}$. Let $\gamma : \mathcal{I} \rightarrow \mathcal{G}$ be a function that assigns a inherent type to each player $i \in \mathcal{I}$, that is, $\gamma(i) = g$ for some $g \in \mathcal{G}$. A player with inherent type g is characterized by a function σ_g that associates to each level of private good consumption a crowding type which is a probability measure over the set of crowding characteristics, that is, the crowding map for player i is,

$$\sigma_{\gamma(i)} : \begin{array}{l} \mathcal{B} \rightarrow \mathcal{P}(\mathcal{C}) \\ x \rightarrow \sigma_{\gamma(i)}(x) \end{array} .$$

The use of probability measures to define a crowding type has the natural interpretation that players are characterized by a mix of crowding characteristics. One interpretation is that different characteristics have different weights. For example, a high weight on “general practitioner of medicine” and a low weight on “brain surgeon” may mean that a player, through his choice of consumption bundle, will, with high probability, be a general practitioner and, with low probability, a brain surgeon. Notice that there could be several types of general practitioners (and several types of brain surgeons). Through his private consumption, while the player may aim to be a brain surgeon and may succeed, with some probability, he may end up being a general practitioner of medicine.

We assume that there is a given set of clubs, which we may think of as locations. Let $\mathcal{S} = \{s^1, \dots, s^K\}$ be the set of clubs and let \mathcal{Y} be the set of public projects. Each club can produce a public project; let y^k denote a

public project produced at location s^k and let \mathcal{Y} denote the set of possible public projects, taken as the same for all clubs $k = 1, \dots, K$. Assume that one possibility is to produce zero output; in this case, if the club has any members, it will be purely social. Also, note that public project levels are not constrained to be finite in number.

As usual in differentiated crowding models we assume that the utility of a player is affected (positively or negatively) by the crowding profile of his club of membership; thus we need to consider the crowding profile of the clubs to which the player might belong. Given the crowding map $\sigma_{\gamma(i)}(\cdot)$ for each player $i \in N$, that depends on the private consumption, let $\mathbf{cp}(s^k) = \sum_{i \in s^k} \sigma_{\gamma(i)}(\cdot)$ be the measure on the set of crowding characteristics obtained by adding the weights assigned to each crowding characteristic over all the players in the club s^k .² Let us denote by $\mathcal{M}(\mathcal{C})$ the set of all finite measures with support in \mathcal{C} ; then $\mathbf{cp}(s^k)(\cdot) \in \mathcal{M}(\mathcal{C})$.

A taste type t is described by an endowment of private good $w_t \in \mathcal{B}$ and by a preference relation \succeq_t defined over $\mathcal{B} \times \mathcal{Y} \times \mathcal{P}(\mathcal{C}) \times \mathcal{M}(\mathcal{C})$.³ Let us denote a consumption bundle as $(x, y, \sigma_g(x), \mathbf{cp})$, where x is a bundle of private goods, y is a club project, $\sigma_g(x)$ is the corresponding crowding type and \mathbf{cp} is the crowding profile of the jurisdiction in which the player resides. We assume that the preference relation of a player of taste type t is represented by an utility function,

$$u_t : \mathcal{B} \times \mathcal{Y} \times \mathcal{P}(\mathcal{C}) \times \mathcal{M}(\mathcal{C}) \longrightarrow \mathbb{R}_+ \\ (x, y, \sigma_g(x), \mathbf{cp}) \rightarrow u_t(x, y, \sigma_g(x), \mathbf{cp}).$$

In addition to affecting utilities, the crowding profile of a club also affects production. The production technology is given by the club cost function,

$$f : \mathcal{Y} \times \mathcal{M}(\mathcal{C}) \rightarrow \mathcal{B},$$

²Note that in this formulation, as in some of the labor market literature (cf., Heckman and Sedlacek 1985), it is only the sum of the characteristics of each player that is relevant. By restricting to a finite set of crowding types, however, we could treat the case where not only the total amount of each characteristic that is present matters but also how that characteristic is distributed among club members.

³In interpretation, it is intuitive to think of endowments as consisting of money and leisure since these are two commodities that are frequently donated by individuals to clubs. But one could imagine that players own other commodities that can either be consumed or contributed to a club, or both.

where $f(y, \mathbf{cp})$ is the cost in terms of private goods of carrying out a club project y for a jurisdiction under crowding profile \mathbf{cp} .

2.2 The non-cooperative game

Crowding type matters for two reasons: (1) Each player has preferences about his own crowding type and (2) the crowding profile of the club to which he belongs affects a player's welfare directly through his preferences and indirectly through the club cost function. Crowding type is viewed as a *consequence* of private consumption and is acquired *before* a player joins a club. Thus, we model the game as one with two stages.

We consider a Nash game with two stages with observed actions, that is, in the second stage all players know the actions chosen at the first stage. All players move simultaneously in each stage. In the first stage, players choose private consumptions, which, given the relationship between private consumption and crowding type, entails the simultaneous choice of crowding types. In the second stage, with private consumptions and crowding types already fixed, players choose clubs.

In the first stage the strategy set for each player i is his consumption set, that is, the set of all levels of private consumption allowable for the player i given his endowment; let us denote this set by $W_{\tau(i)} = \{b \in \mathcal{B} : b \leq w_{\tau(i)}\}$. Thus, a profile of strategies in the first stage of the game is a vector of private consumptions, $(x_1, \dots, x_I) \in W_{\tau(1)} \times \dots \times W_{\tau(I)}$. Given that the choice of private consumption means also the choice of the crowding type through the crowding map, the vector of private consumptions determines the crowding types of all players in the game; $(\sigma_{\gamma(1)}(x_1), \dots, \sigma_{\gamma(I)}(x_I)) \in \mathcal{P}(\mathcal{C}) \times \dots \times \mathcal{P}(\mathcal{C})$.

In the second stage of the game, given the history of stage 1 (which implies that the crowding profile of the population is fixed), each agent chooses one club to join from the entire set of clubs, $\mathcal{S} = \{s^1, \dots, s^K\}$.

The contribution of player i to funding the costs of the club good is given by the surplus of his endowment over his private consumption, $w_{\tau(i)} - x_i$. Therefore, we assume that a club s^k with crowding profile $\mathbf{cp}(s^k)$ uses no more than the available bundle of private goods $\sum_{i \in s^k} (w_{\tau(i)} - x_i)$ to produce the club good. Thus, if club good y^k is produced in club s^k with crowding

profile $\mathbf{cp}(s^k)$ then the inequality:

$$f(y^k, \mathbf{cp}(s^k)) \leq \sum_{i \in s^k} (w_{\tau(i)} - x_i)$$

must be satisfied. We assume that the amount of club good produced is the largest possible given the inequality constraint; this is consistent with monotonicity of preferences for the public good.

A sequence of strategies for the two stages is a profile of private consumptions $X = (x_1, \dots, x_I)$ in the first stage and an assignment of individuals to clubs in the second stage, $S = (s_1, \dots, s_I)$, where $s_i \in \mathcal{S}$ denotes the club choice of player i . Each assignment of players to clubs determines a partition $\{s^k\}^4$ of players among the clubs in $S = (s^1, \dots, s^K)$, where $s^k = \{i \in \mathcal{I} : s_i = s^k\}$. The payoff of player i is defined by his utility function evaluated over a strategy profile for both stages, $(x_1, \dots, x_I, s_1, \dots, s_I)$, as follows,

$$U_i : W_{\tau(1)} \times \dots \times W_{\tau(I)} \times \mathcal{S} \times \dots \times \mathcal{S} \longrightarrow \mathbb{R}$$

$$U_i(x_1, \dots, x_I, s_1, \dots, s_I) = u_{\tau(i)}(x_i, y^k, \sigma_{\gamma(i)}(x_i), \mathbf{cp}(s^k)).$$

Let $\mathcal{G} \equiv \{(W_{\tau(i)}, \mathcal{S}), U_i; i = 1, \dots, I\}$ denote the two stage game just defined. This two stage game can also be described as a game in strategic form; let $\bar{\mathcal{G}} \equiv \{W_{\tau(i)} \times \mathcal{S}, U_i; i = 1, \dots, I\}$ denote this strategic game where $W_{\tau(i)} \times \mathcal{S}$ is the strategy set of the player i and U_i is the payoff function of the player i .

2.3 Equilibrium: definitions and main results

Definition. A strategy profile $(X, S) \in W_{\tau(1)} \times \dots \times W_{\tau(I)} \times \mathcal{S} \times \dots \times \mathcal{S}$ is an *equilibrium* for the economy if it is a Nash equilibrium for the two-stage game in the normal form $\bar{\mathcal{G}}$.

Definition. A strategy profile $(X, S) \in W_{\tau(1)} \times \dots \times W_{\tau(I)} \times \mathcal{S} \times \dots \times \mathcal{S}$ is a *strategic club equilibrium* for the economy if it is a subgame-perfect equilibrium for the two-stage game \mathcal{G} .

⁴Note that this is perhaps a slight abuse of terminology since one or more members of the partition may be empty.

Theorem 1. There exists an equilibrium in mixed strategies for the economy.

Proof. The strategy set of each player is finite; therefore by Nash (1950) there exists a mixed strategy equilibrium for the game.

Theorem 2. There exists a strategic club equilibrium in mixed strategies for the economy.

Proof. Let us show that there exists a subgame perfect equilibrium for the sequential game $\mathcal{G} \equiv \{(W_i, \mathcal{S}), U_i; i = 1, \dots, I\}$, that is, we need to show that there exists a Nash equilibrium that is a Nash equilibrium for every subgame of the original game. We use backward induction. First we state that for any given strategy in the first period, the subgame of the second stage has an equilibrium. In fact, given a vector of private consumption in the first stage, $X = (x_1, \dots, x_I)$, the subgame that we obtain is the game in normal form $\mathcal{G}^X \equiv \{(\mathcal{S}, U_i^X); i = 1, \dots, I\}$ where the strategy set for each player is the set of clubs, \mathcal{S} , and the payoff function is, $U_i^X(s_1, \dots, s_I) = u_{\tau(i)}(x_i, y^k, \sigma_{\tau(i)}(x_i), \mathbf{cp}(s^k))$.

The strategy set for the subgame \mathcal{G}^X is finite; therefore we can apply the result of Nash (1950) to conclude that there exists a Nash equilibrium in mixed strategies for every game \mathcal{G}^X . Now, we go back to the first stage and consider the following game. In the first stage each player i chooses a level of private consumption, $x_i \in W_{\tau(i)}$. For each profile of private consumptions $X = (x_1, \dots, x_I)$ all the players expect the same second-stage Nash equilibrium, let $P = (p_1, \dots, p_I)$ be an equilibrium in mixed strategies for the subgame \mathcal{G}^X . The payoff for player i in the first stage is defined as $\Pi_i(X) = Exp_P U_i(X, \mathcal{S})$. Once again we have a finite game so there exists an equilibrium in mixed strategies for the first stage of the game by Nash (1950), denote the mixed strategy equilibrium of the first stage of the game by \mathcal{X} . The profile (\mathcal{X}, P) is a subgame perfect equilibrium for the sequential game \mathcal{G} and therefore is a strategic club equilibrium for the economy.

2.4 An illustration of mixed strategies for the second stage

In a mixed strategy equilibrium of the second period each player chooses a probability distribution over clubs, that is, in the same spirit of Garret and Qin (1996) players chose lotteries, but here the lotteries are over the finite set of clubs. This is illustrated below by an example.

Exemplo 1:

Let $\mathbb{S} = \{S = (s^1, \dots, s^K) : S \text{ is a partition of the total player set } \mathcal{I}\}$, be the set of all possible partitions of players among the K jurisdictions.

We denote by $P = (p_1, p_2, \dots, p_I) \in \Delta^K \times \dots \times \Delta^K = (\Delta^K)^I$ a profile of individual player lotteries p_i over clubs. Each profile $P = (p_1, p_2, \dots, p_I)$ gives rise to a joint lottery L over the set \mathbb{S} . Thus, individual player lotteries p_i are marginal distributions of the joint lottery L . These concepts are illustrated in the following example.

Let $I = 3$ and $K = 2$. Suppose that players 1 and 2 have crowding type with total mass in c and player 3 has crowding type with total mass in c' . Consider the following individual lotteries over clubs:

Individual Lotteries		
Clubs	s_1	s_2
Probabilities	p_1	p_2
player 1	$\frac{1}{2}$	$\frac{1}{2}$
player 2	$\frac{1}{3}$	$\frac{2}{3}$
player 3	$\frac{1}{4}$	$\frac{3}{4}$

Then the corresponding joint lottery is given by,

Joint Lottery - L		
Potential clubs- \mathbb{S}	Crowding Profile	Probability
$(\emptyset, \{1, 2, 3\})$	$(\emptyset, \{c, c, c'\})$	$\frac{6}{24}$
$(\{1\}, \{2, 3\})$	$(\{c\}, \{c, c'\})$	$\frac{6}{24}$
$(\{2\}, \{1, 3\})$	$(\{c\}, \{c, c'\})$	$\frac{3}{24}$
$(\{3\}, \{1, 2\})$	$(\{c'\}, \{c, c\})$	$\frac{2}{24}$
$(\{1, 2\}, \{3\})$	$(\{c, c\}, \{c'\})$	$\frac{3}{24}$
$(\{1, 3\}, \{2\})$	$(\{c, c'\}, \{c\})$	$\frac{2}{24}$
$(\{2, 3\}, \{1\})$	$(\{c, c'\}, \{c\})$	$\frac{1}{24}$
$(\{1, 2, 3\}, \emptyset)$	$(\{c, c, c'\}, \emptyset)$	$\frac{1}{24}$

Let

$$\mathbb{S}_i^k = \{s^k : i \in s^k\},$$

denotes the set of potential allocations of players to club s^k that contains the player i . In Example 1, $\mathbb{S}_1^1 = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$.

As usual in differentiated crowding models we assume that the utility of a player is affected (positively or negatively) by the profile of crowding characteristics of the players living in his jurisdiction, thus, we need to consider the crowding profile of the clubs to which the player might belong.

Let $\mathcal{CP}(\mathbb{S}_i^k)$ denotes the corresponding set of potential crowding profiles of the club s^k conditional on player i belonging to club s^k . In Example 1, $\mathcal{CP}(\mathbb{S}_1^1) = \{\{c\}, \{c, c\}, \{c, c'\}, \{c, c, c'\}\}$. We denote the elements of the set \mathbb{S}_i^k by s_i^{kh} for $h = 1, \dots, \#\mathbb{S}_i^k$ and the corresponding crowding profile by $\mathbf{cp}(s_i^{kh})$ for $h = 1, \dots, \#\mathbb{S}_i^k$.

The joint lottery, L , induce a conditional distribution over \mathbb{S}_i^k , that we denote by $l_i^{s^k}$, that is, $l_i^{s^{kh}}$ denotes the probability of the allocation s_i^{kh} for the jurisdiction s^k given that $i \in s^k$, given by the distribution $l_i^{s^k}$. In summary, a given allocation of individual lotteries $P = (p_1, \dots, p_I)$ induces a joint lottery L over the set of pure allocation of players through jurisdictions and a collection of conditional distributions over jurisdictions that contains the player i , $\left\{l_i = \left(l_i^{s^1}, \dots, l_i^{s^K}\right), i = 1, \dots, I\right\}$ with \mathbb{S}_i^k being the support of the distribution $l_i^{s^k}$ for all $k = 1, \dots, K$ and $i = 1, \dots, I$. In the following three tables we describe the distributions $l_1^{s^1}$, $l_2^{s^1}$, $l_3^{s^1}$, $l_1^{s^2}$, $l_2^{s^2}$ and $l_3^{s^2}$ for the example 1.

\mathbb{S}_1^1	$\mathcal{CP}(\mathbb{S}_1^1)$	$l_1^{s^1}$	\mathbb{S}_2^1	$\mathcal{CP}(\mathbb{S}_2^1)$	$l_2^{s^1}$	\mathbb{S}_3^1	$\mathcal{CP}(\mathbb{S}_3^1)$	$l_3^{s^1}$
{1}	{c}	$\frac{6}{12}$	{2}	{c}	$\frac{3}{8}$	{3}	{c'}	$\frac{2}{6}$
{1, 2}	{c, c}	$\frac{3}{12}$	{1, 2}	{c, c}	$\frac{3}{8}$	{1, 3}	{c, c'}	$\frac{2}{6}$
{1, 3}	{c, c'}	$\frac{2}{12}$	{2, 3}	{c, c'}	$\frac{1}{8}$	{2, 3}	{c, c'}	$\frac{1}{6}$
{1, 2, 3}	{c, c, c'}	$\frac{1}{12}$	{1, 2, 3}	{c, c, c'}	$\frac{1}{8}$	{1, 2, 3}	{c, c, c'}	$\frac{1}{6}$

\mathbb{S}_1^2	$\mathcal{CP}(\mathbb{S}_1^2)$	$l_1^{s^2}$	\mathbb{S}_2^2	$\mathcal{CP}(\mathbb{S}_2^2)$	$l_2^{s^2}$	\mathbb{S}_3^2	$\mathcal{CP}(\mathbb{S}_3^2)$	$l_3^{s^2}$
{1, 2, 3}	{c, c, c'}	$\frac{6}{12}$	{1, 2, 3}	{c, c, c'}	$\frac{3}{8}$	{1, 2, 3}	{c, c, c'}	$\frac{2}{6}$
{1, 3}	{c, c'}	$\frac{3}{12}$	{2, 3}	{c, c'}	$\frac{3}{8}$	{2, 3}	{c, c'}	$\frac{2}{6}$
{1, 2}	{c, c}	$\frac{2}{12}$	{1, 2}	{c, c}	$\frac{1}{8}$	{1, 3}	{c, c'}	$\frac{1}{6}$
{1}	{c}	$\frac{1}{12}$	{2}	{c}	$\frac{1}{8}$	{3}	{c'}	$\frac{1}{6}$

2.5 An illustration of probability measures over the set of crowding characteristics

The next example illustrates two main features that arise in our model as a consequence of taking a crowding type as a probability measure over the

set of crowding characteristics. First, a player can be characterized by both exogenous and endogenous crowding characteristics; second, for a specific crowding characteristic the player can be characterized by different levels of that crowding characteristic and in some situations this conforms to reality.

Exemplo 2:

Let the set of crowding characteristics be: {Male, Female, Good Appearance, Bad Appearance, Good Skills, Bad Skills}= $\{M, F, GA, BA, GS, BS\}$.

There are four inherent types, $\{g_1, g_2, g_3, g_4\}$ which means that there are four different crowding maps that we describe as follows,

$$c_{g_1}(x) = 0.2M + \alpha_{g_1}^1(x)GA + \alpha_{g_1}^2(x)BA + \beta_{g_1}^1(x)GS + \beta_{g_1}^2(x)BS,$$

$$c_{g_2}(x) = 0.2M + \alpha_{g_2}^1(x)GA + \alpha_{g_2}^2(x)BA + \beta_{g_2}^1(x)GS + \beta_{g_2}^2(x)BS$$

$$c_{g_3}(x) = 0.2F + \alpha_{g_3}^1(x)GA + \alpha_{g_3}^2(x)BA + \beta_{g_3}^1(x)GS + \beta_{g_3}^2(x)BS,$$

$$c_{g_4}(x) = 0.2F + \alpha_{g_4}^1(x)GA + \alpha_{g_4}^2(x)BA + \beta_{g_4}^1(x)GS + \beta_{g_4}^2(x)BS,$$

$$\text{with } \alpha_{g_l}^1(x) + \alpha_{g_l}^2(x) = 0.4 \text{ and } \beta_{g_l}^1(x) + \beta_{g_l}^2(x) = 0.4 \text{ for all } l = 1, 2, 3, 4.$$

As we can see the probability measure in the crowding characteristics is divided between gender, appearance and skills. The weights on gender do not depend on consumption of private good x , which means that gender is an exogenous crowding characteristic that can appear as an aspect of an agent's crowding type. The weights on GA and BA describe the appearance, better appearance means higher $\alpha_{g_l}^1(x)$ and lower $\alpha_{g_l}^2(x)$ and the opposite means bad appearance. The same for skills. The weights on GS and BS describes skills and more skilled means higher $\beta_{g_l}^1(x)$ and lower $\beta_{g_l}^2(x)$ and the opposite means less skilled.

If we assume for the inherent types g_1 and g_2 that for a given a level of private consumption x , $\alpha_{g_1}^1(x) < \alpha_{g_2}^1(x)$, $\alpha_{g_1}^2(x) > \alpha_{g_2}^2(x)$ and $\beta_{g_2}^1(x) < \beta_{g_2}^1(x)$, $\beta_{g_1}^2(x) > \beta_{g_2}^2(x)$, then, it could be that the inherent type g_2 is more desirable than inherent type g_1 , because for the same level of private consumption the player with inherent type g_2 have more weight in GA and in GS (which also means less weight in BA and in BS).

The inherent types g_3 and g_4 are different from the inherent types g_1 and g_2 because g_3 and g_4 are female and g_1 and g_2 are male. But they also can

be different because the weight functions are different. For instance, if for a given level of private consumption x , $\alpha_{g_3}^1(x) < \alpha_{g_4}^1(x)$, $\alpha_{g_3}^2(x) > \alpha_{g_4}^2(x)$ and $\beta_{g_3}^1(x) < \beta_{g_4}^1(x)$, $\beta_{g_3}^2(x) > \beta_{g_4}^2(x)$, then, the inherent type g_4 is more desirable for club formation than the inherent type g_3 . Note that we are supposing that for the purposes of clubs good appearance and good skills are desirable.

3 Equilibrium discussion

As stressed in the introduction, in a strategic club equilibrium a player chooses private consumption taking into account the utility that he can obtain in the future from club membership and the attractiveness of a player to a club depends on his contribution of private goods and on his crowding type. His utility also depends on scarcity of players of that type. By taking into account these effects we obtain a model that is able to describe and analyze the outcomes of such features. The nature of strategic interactions, however, apparently limits the possibility of general comparative statics results. In fact, in different contexts the same forces that drive the model can have opposite effects, leading to different results. In the following subsections we present examples illustrating that in fact the properties of the model depend mainly on the properties of the utility functions and on relative scarcities of player types.

3.1 The effects of the relationship between private consumption and crowding type

In our model, the characteristics of players that matter inside a club – crowding types and the amount of private good than a player contributes – are interpreted as observable by other club members. Another feature of the model that we wish to stress is that since crowding types are the characteristics that matter inside a club and these characteristics are the result of the private consumption, sometimes a player has to make a trade-off between private consumption and a more advantageous crowding type. The dependence of crowding type on private consumption effectively creates an ‘externality’ but, unlike the usual situation with externalities as usually described, this externality is internal to the player. The effects of the externality are strongly

influenced by the exogenous relationship between private consumption and crowding type.

This example illustrates these two features.

Example 3:

Let the crowding characteristics be {secondary school, undergraduate}= $\{SC,UG\}$ and suppose that the admissible probability distribution over types assigns probability to one or the other crowding type.

Suppose every player has the same inherent type g and therefore the same crowding map,

$$\sigma_g(x) = \begin{cases} SC & \text{if } x \leq x' \\ UG & \text{if } x > x' \end{cases} .$$

These population forms clubs for social reasons, for example to exchange and consume music. Let us assume that there are two taste types, t_1 and t_2 . Taste type t_1 prefer jazz, and prefer to be in clubs with undergraduate players because they expect that people with undergraduate level education prefer jazz to country and western and will be more valuable for the purpose of organize concerts, change CD and discuss music. Players with taste type t_2 by other side prefer country and western music and prefer to be in clubs with players with secondary school level because they expect that these players with high probability like country and western music. Let us assume that the private good is study time or money spend on school school supplies and we suppose that for both taste types the utility is decreasing with the private good (players do not like to study or do not like spend money in school supplies).

Assume that preferences are represented by the following utilities:

Club	Utility t_1	Utility t_2
SC	$A - 2x$	$B + 10 - x$
UG	$A + 10 - 2x$	$B - x$
SC,UG	$A + 5 - 2x$	$B + 5 - x$

The first column denotes the kind of club to which the player belongs, that is, whether the club has only players with secondary level education or only players with undergraduate education or players with both kinds of crowding types, respectively. In these utilities the attractiveness of a player to a club is essentially the crowding type. We assume that there are at least three club locations, so that all three sorts of clubs can exist.

Assume that both types of players have the same endowment satisfying $w_t > x'$; then all players have sufficient endowment to become undergraduates. Let x denote the private consumption, assuming that taste type t_1 dislikes studying more than taste type t_2 . The numbers A and B are constants sufficiently large to ensure that the utility is always nonnegative. If $x' = 5$, players with taste type t_1 will be better off if they consume x' , become undergraduates, and join a club consisting only of undergraduate players. Players of type t_2 will consume $x = 0$, obtain just secondary school education, and in the second stage they will belong to clubs with consisting of players with secondary school education only. Thus no one inside a club is concerned about how much a player dislikes studying; only the crowding type of a player is relevant. , they just care about the crowding type of each player. In this example we can also observe that the player with taste type t_1 do not like the private consumption, actually he like it less than taste type t_2 , but since their utility puts more weight on the club good than on private consumption, the players sacrifice the private consumption to obtain their most suitable crowding type for the purpose of club membership.

The players with taste type t_2 do not need to give up private consumption to obtain their best crowding type.

3.2 Changes in the inherent type distribution

In our model crowding type is a consequence of private consumption and we assume that players have different aptitudes to acquire crowding types. This aptitude is described by the player's inherent type. The next examples illustrate effects of this feature on club formation, namely, how changes in the distribution of the inherent types will affect the crowding profile of the economy and therefore the attainable clubs and the welfare of the players.

Example 4:

As above, we suppose that the admissible probability distribution over types assigns probability to one or the other crowding characteristic. The crowding types are: {doctor, house cleaner }= $\{D, HC\}$.

Inherent types: $\{g_1, g_2\}$ with

$$\sigma_{g_1}(x) = \begin{cases} HC & \text{if } x \leq \underline{x} \\ D & \text{if } x > \underline{x} \end{cases}, \quad \sigma_{g_2} = \begin{cases} HC & \text{if } x \leq \bar{x} \\ D & \text{if } x > \bar{x} \end{cases} \quad \text{and } \underline{x} < \bar{x}.$$

Higher private consumption, denoted by x , is interpreted as more time spent studying. Here utility will be decreasing in private consumption (assuming players do not like studying). There are three kinds of clubs, clubs with only doctors, clubs with only house cleaners and clubs with both doctors and house cleaners. We assume that there are at least three club locations, so that all three sorts of clubs can exist. There is just one taste type in the economy, described as follows,

Club	Utility D	Utility HC
D	$16 - x$	-
HC	-	$5 - x$
$D + HC$	$18 - x$	$7 - x$

Suppose that $\underline{x} = 5 < w_t = 8 < \bar{x} = 10$. Thus, all players with inherent type g_2 will be house cleaners, will not consume private good and will obtain utility 5 or 7. A player with crowding type g_1 could decide to refrain from consumption of the private good and obtain utility 5 or 7. Alternatively, he could consume sufficient private commodities to achieve the crowding type of doctor, and, in that case, he obtain utility 11 or 13. Therefore, all players with crowding type g_1 will consume the private good in order to obtain the crowding type that realizes a higher utility.

If we increase the number of players with the inherent type g_1 , there will be more doctors in the economy. This means that there will be more clubs of type D and $D + HC$, which implies that the payoff of some players with inherent type g_1 will decrease but the payoff of some players with inherent type g_2 could increase. If we increase the number of players with inherent type g_2 , the number of house cleaners will increase, leading to an increase in the number of clubs of type HC and $D + HC$. In consequence, the payoffs of some players with inherent type g_2 will decrease but the payoff of some players with inherent type g_1 could increase.

Now, we examine the consequences of trying to manipulate the inherent type. If we modify his endowment, we can change the ability of an agent to acquire a crowding type. We observe that if we subsidize players of one inherent type this may increase their welfare but may also decrease the welfare of the players with the other type. Actually, as opposed to the previous example, in this example, there are complementarities between players of different crowding types; therefore an increase in the scarcity of one crowding

type will lead to a welfare decrease for the other type.

Suppose that players of inherent type g_2 are subsidized so that their endowment (post subsidy) becomes $w_t = 10$. Thus, these players could choose to become doctors by spending all their endowment and their utility will be 8 or 10 or they could choose to be an house cleaner and obtain 5 or 7. Also, all the players with inherent type g_2 will become doctors. But then in this economy all the players will be doctors. The players with inherent type g_1 , initially in clubs with house cleaners, will be worse after the subsidy, while the players of type g_2 will all be better after the subsidy.

Example 5:

In this example we again examine consequences of inherent type changes and consequent crowding type changes. This is a context different from the previous example and the conclusions reflect that difference. The example is formulated so that inherent type and crowding type are the same and one crowding type is not desirable to players of the other type; that is, there are no complementarities between different types. All players, however, prefer to be in two-person clubs, even if they must share the club with a less-desired partner.

We assume:

Crowding types: {smoker, no smoker }= $\{S, NS\}$.

Inherent types: $\{g_1, g_2\}$ with crowding maps,

$$\sigma_{g_1}(x) = \begin{cases} NS & \text{if } x \leq \underline{x} \\ S & \text{if } x > \underline{x} \end{cases}, \quad \sigma_{g_2} = \begin{cases} NS & \text{if } x \leq \bar{x} \\ S & \text{if } x > \bar{x} \end{cases} \text{ and } \underline{x} < \bar{x}.$$

Preference types: {indifferent (to smoke), almost hater (or smoke)}= $\{I, AH\}$. The utility received by a player who is in a club with two players:

$U_I(NS; NS) = 5$	$U_{AH}(NS; NS) = 10$
$U_I(NS; S) = 5$	$U_{AH}(NS; S) = 5$
$U_I(S; NS) = 5$	$U_{AH}(S; NS) = 0$
$U_I(S; S) = 5$	$U_{AH}(S; S) = 0$

The first coordinate in the utility function U_i denotes the crowding type of the player with that utility function (where i is either I or AH) and the second coordinate denotes the crowding type of the other player in the club.

Let there be 100 players of each of all four possible taste and inherent types: Ig_1 , Ig_2 , AHg_1 and AHg_2 . We assume that there are sufficiently many clubs so that all players can be accommodated in clubs with only two people, and that being in a three-person club is less desirable than being alone.

We assume that all players have the same endowment and consume their entire endowment. We suppose that $\underline{x} < w_t < \bar{x}$. Then, the players with inherent type g_1 will all be smokers and the players with inherent type g_2 will be non-smokers.

Suppose we increase by 100 the number of players of inherent type Ig_2 in the economy. This means that we increase by 100 the number of non-smoking players in the economy; then the payoffs of the indifferent players are unaffected but the payoffs of the almost hater players could increase.

Suppose instead that we increase by 100 the number of players of inherent type Ig_1 in the economy. This means that we increase by 100 the number of smokers in the economy; then the payoff of the indifferent players is unaffected but the payoff of the almost hater players could decrease.

In this example there are no complementarities between players of different crowding types, in fact the scarcity of the smoker crowding type implies a welfare increase, however the scarcity of the nonsmoker crowding type implies a welfare decrease.

3.3 Distribution of crowding types across clubs

Concerning the distribution of crowding types across clubs, we can observe that the result once more depends on the special context. Indeed, in example 3 in a subgame perfect equilibrium there is segregation of the players by crowding types. If a player with crowding type SC is in a club where all players have secondary level education then the player can not increase his utility by choosing another club; the same is true for the players with crowding type UG . Moreover, at equilibrium the players are also segregated by taste types. In contrast, in example 4 the opposite is true, that is, at equilibrium there is a mix of crowding types in each club. The main source of that outcome is that the crowding types are complementaries for the provision of the club good. In this example a player is better if he belongs to a club with both kind of crowding type than if he belongs to a club where the other players have the same crowding type as him.

4 Matching models: A special case where we can avoid mixed strategies in the second stage

In this section we consider a special setting of our model and then we apply an equilibrium concept from Gale and Shapley (1962) and the main result is that for this special case we can avoid mixed strategies over the clubs.

The players choose private consumption. Given the private consumption, the crowding map determines the crowding type of each player. Let us consider a finite (with more than one element) set of crowding characteristics, and we assume also that at least one of the crowding characteristic is exogenous. Let us label it as Male (M) or Female (F). But this fixed exogenous crowding characteristic could be any other. The remaining crowding characteristics could be endogenous or exogenous. The point is that we will consider clubs with only one or two players; in a two-player club, one player must be Male and the other must be Female. We assume that there is no club good; that is, the club good level is zero in all clubs. What affects the payoff of each player is the crowding type of the other player inside the club. We assume that for any given crowding profile of all players, the preferences of each player with exogenous crowding characteristic F are such that he can rank all the M players. The same is true for those with characteristic M ; their preferences provide rankings over all possible crowding types of F players. To resume, in this particular setting the objective in the second stage is to match a Male with a Female (if such matchings are advantageous) and the payoff or utility that each agent obtain in each club depends only on the crowding type of the other player inside the club.

Actually, we have a matching game in the second stage and given the assumptions on preferences, we are in conditions to apply the “deferred acceptance” approach from Gale and Shapley (1962) and therefore we can guarantee the existence of a stable set of clubs. This equilibrium concept is defined in Gale and Shapley (1962). If we suppose that what players expect in the second stage is the stable set of clubs equilibrium concept instead of a Nash equilibrium then we do not need to use mixed strategies in the second stage. In the first stage the players choose private consumption and obtain a crowding type; for the second stage the agents anticipate a stable set of clubs, and each player evaluates the payoff that he obtain in the club to which he belongs with certainty. Then, given these payoffs, the agents play a Nash

game in the first stage to choose the private consumption. The example 2 introduced in the subsection 3.5 can help to understand this particular case.

In research in progress, we formalize the claims of this subsection and extend our results to demonstrate conditions under which there exists a club equilibrium in pure strategies. We also illustrate how the results of Kalai (2004) can be applied to obtain ex-post approximate equilibrium in pure strategies.

5 Related Literature

This work is related to Konishi, Le Breton and Weber (1998) and others in the sense that a noncooperative framework is used to assign consumers to clubs for the purpose of collective consumption within clubs. Konishi, Le Breton and Weber use a Nash game with only one stage; instead, we use a two stage game. Our dynamic framework is motivated by the feature that crowding types may be determined prior to club memberships – acquisition of skills, for example, is not easily changed and may be relatively fixed when one enters the job market. Another main difference is the scheme to finance the public good cost. In Konishi, Le Breton and Weber the player pays part of the costs of the public good via a proportional income tax or with a poll tax. In our work we assume that first the players decide on the private consumption level and then they contribute the remainder of their endowment to finance the club good. They may contribute, for example, with skills, wealth, or private commodities. More concretely, for example a player could choose to become a physician or he could retain more of his endowment – monetary wealth and leisure – to contribute to clubs. Moreover, with our equilibrium concept we obtain an intrajurisdictional efficient equilibrium as defined in the paper by Konishi, Le Breton and Weber (1998). Intra-jurisdictional efficiency requires that any group of consumers who are choosing the same club good level are not able to find a different club good level which would make all members of the group better off. In our model each club makes use of the available private good (taking into account indivisibilities) to produce the club good. Thus, there is no way to produce a different level of club good which would make all members of the club better off. In Konishi, Le Breton and Weber, what matters to players is the level of public good and what they have to pay for it, while here crowding type also matters and may affect other agents in complicated ways. Also, having a desirable crowding type may compensate

for inability to make a large contribution of private goods to clubs.

In this work we use the concept of crowding type, that is, observable characteristics that directly affects the welfare of other agents in the same club, introduced in Conley and Wooders (1996,1997) in a cooperative framework. In Conley and Wooders (1996) the crowding types are exogenously given but in Conley and Wooders (1997,2001) the crowding types are endogenously determined in the model, to be precise, the agents buy a crowding type in a set of crowding types. Our work depart from Conley and Wooders framework not only because we consider a strategic approach but also because we consider a different way to bring up the crowding types, namely, we assume that the crowding type is a by product of the private consumption given by an exogenous map that applies private consumptions into crowding types. Our argument is that in many situations the crowding type is actually a consequence of private consumption (well dressed people, smoker people, etc) rather than specific purchase of crowding type as in Conley in Wooders (1997,2001). Another argument in favour of the crowding map is that it can accommodate simultaneously exogenous (gender, age,etc) and endogenous (skills, profession, lifestyle) crowding types. Finally we remark that this original way of bringing up the crowding type is not innocuous in the sense that if we introduce it in the noncooperative framework of Conley and Wooders we have to add some assumptions in order that their results still hold. The point is that a key prove in Conley and Wooders uses a contradiction that basically says that with monotone preferences in the private good one can improve upon a core state by increasing the private consumption of an agent and letting him with the same crowding type. The difficult is that the crowding map do not allow this reasoning unless we add some assumptions because if we change the private consumption of a player his crowding type implicitly could change. Having another crowding type the player could not be any more desirable in the club or could not allow the production of the club good. It is easy to prove that with one of the two following assumptions the argument in Conley and Wooders just described is recovered, however the example 5 above shows that these assumptions doo not necessarily holds.

Assumption 1: *For all inherent types g the function $\sigma_{\mathbf{g}} : \mathfrak{R} \rightarrow \mathcal{C}$ is a step function and for every level of private consumption, $a \in \mathfrak{R}$, $\lim_{x \rightarrow a^+} \sigma_{\mathbf{g}}(x) = \sigma_{\mathbf{g}}(a)$, that is, the function $\sigma_{\mathbf{g}}(\cdot)$ is right continuous.*

This assumption states that certain levels of consumption are required to reach a specific crowding type.

Assumption 2: (a) If $x_1 > x_2$ then every player type of type t prefers crowding type $\sigma_g(x_1)$ to $\sigma_g(x_2)$ for every inherent type g .
(b) If in a jurisdiction we replace a player with crowding type $\sigma_g(x_2)$ by a player with crowding type $\sigma_g(x_1)$ (with $x_1 > x_2$), this new crowding profile of the jurisdiction still able to produce the same club good and the utilities of the players that remain in the jurisdiction do not decrease with this new club crowding profile.

This assumption says that more expensive crowding types are more preferred.

The assumptions asserted have an economic interpretation however it is easy to get crowding maps that do not satisfy these assumptions. In the example 5 of the previous section neither assumption 1, nor assumption 2 are verified. The two crowding maps are neither right continuous nor it is true that more expensive crowding types is more preferred because more private consumption means more smoker people which is not more preferred.

6 Concluding remarks

We set a model that allows to allocate players over clubs in a noncooperative way. With this equilibrium concept we can explore how private consumption can be strategically adjusted by a player so as to take into account the effects of crowding type on his desirability as a member of a club. In this setting the marginal rate of substitution between private good and club good is not just given by preferences, it is also shaped by an exogenous relation between private consumption and crowding type which is given by the crowding map. In fact, this exogenous relation incorporates several effects that players are not able to manipulate but are decisive to define their external effects. We stress two of them, the ability of a player to acquire a given crowding type from the private consumption that depends on the inherent type and the way that society values different crowding characteristics resulting from private consumption. For example, in a more liberal society (a cosmopolitan city) the crowding map could be different from that of a more conservative society (a country or small city) even for individuals of the same inherent type.

Moreover, after the private consumption and consequently the crowding type are fixed, what matters to join a club are the two very different features of a player, his ability to pay for the costs of the club good, the remaining

of his endowment, and his crowding type. These two features could work as substitutes, for example, it could happen that a player with not much endowment is in the same club of a rich player because he has a good inherent type that allows him to obtain a desirable crowding type even with few private consumption. However, substitution is not necessarily true, for instance, to be a rich player is not enough for a player to join any club he wants just because he has enough money to pay for the club good. In fact, if he has a non desirable crowding type no one wants to be with him in the same club.

Finally we emphasize that there are other fields where the crowding type could play a major role. For instance, Demange (2005) explores how competition works in an economy where two opposite forces act together, namely, increasing returns to scale and individuals preferences over diversity. Our claim is that these results could be refined if the crowding type concept is included in her competitive framework. In fact, these two forces are also present in our framework and it is clear how both forces are affected by the crowding type. When the crowding type describes skills this means that the crowding type affects increasing returns to scale, when the crowding type is for example smoker or well dressed people, then crowding type is mainly affecting the individual preferences over diversity. Moreover, in our framework the private consumption determines the crowding type, then, we obtain that the balance between increasing returns and individual preferences over the club good are indirectly shaped by the private consumption through the crowding type.

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