ERRATUM : MESH ADAPTIVE DIRECT SEARCH ALGORITHMS FOR CONSTRAINED OPTIMIZATION*

CHARLES AUDET†, A. L. CUSTÓDIO‡, AND J. E. DENNIS, JR.§

Abstract. In [SIAM J. Optim., 17 (2006), pp. 188-217] Audet and Dennis proposed the class of *mesh adaptive direct search algorithms* (MADS) for minimization of a nonsmooth function under general nonsmooth constraints. The notation used in the paper evolved since the preliminary versions and, unfortunately, even though the statement of Proposition 4.2 is correct, its proof is not compatible with the final notation. The purpose of this note is show that the proposition is valid.

Key words. mesh adaptive direct search algorithms (MADS), constrained optimization, nonsmooth optimization

AMS subject classifications. 90C30, 90C56, 65K05, 49J52

In [1] Audet and Dennis proposed the class of *mesh adaptive direct search algorithms* (MADS) for minimization of a nonsmooth function under general nonsmooth constraints. The paper contains a convergence analysis for this class of methods, and proposes two variants of an implementable instance called LTMADS.

The proof that LTMADS is indeed an instance of MADS is not compatible with the notation used in the rest of the paper. We restate the proposition and propose a consistent proof.

PROPOSITION 0.1 (Proposition 4.2 of [1]). *At each iteration k, the procedure above yields a D_k and a MADS frame P_k such that*

$$P_k = \{x_k + \Delta_k^m d : d \in D_k\} \subset M_k,$$

where M_k is given by Definition 2.1 *and D_k is a positive spanning set such that for each $d \in D_k$,*

- *d can be written as a nonnegative integer combination of the directions in D: $d = Du$ for some vector $u \in \mathbb{N}^{n_D}$ that may depend on the iteration number k;*
- *the distance from the frame center x_k to a frame point $x_k + \Delta_k^m d \in P_k$ is bounded above by a constant times the poll size parameter: $\Delta_k^m \|d\|_\infty \leq \Delta_k^p \max\{\|d'\|_\infty : d' \in D\}$;*
- *limits (as defined in Coope and Price* [2]*) of convergent subsequences of the normalized sets $\overline{D_k} := \{\frac{d}{\|d\|_\infty} : d \in D_k\}$ are positive spanning sets.*

Proof. In order to construct the set of directions D_k, the algorithm builds matrices at iteration k that should be called L_k, B_k and B'_k. To ease the presentation, we omit the index k in the proof of the two first bullets. The index k reappears in the proof of the last bullet since this last result involves limits as k goes to infinity.

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†GERAD and Département de Mathématiques et de Génie Industriel, École Polytechnique de Montréal, C.P. 6079, Succ. Centre-ville, Montréal (Québec), H3C 3A7 Canada (Charles.Audet@gerad.ca, http://www.gerad.ca/Charles.Audet).

‡Departamento de Matemática, FCT-UNL, Quinta da Torre, 2829-516 Caparica, Portugal (alcustodio@fct.unl.pt, http://ferrari.dmat.fct.unl.pt/personal/alcustodio/).

§Computational and Applied Mathematics Department, Rice University, 8419 42nd Ave SW, Seattle, WA 98136 (dennis@rice.edu, http://www.caam.rice.edu/∼dennis).

By the construction in [1], L is a lower triangular $(n-1)\times(n-1)$ matrix where each term on the diagonal is either plus or minus 2^ℓ, and the lower components are randomly chosen from the discrete set $\{-2^\ell+1, -2^\ell+2, \dots, 2^\ell-1\}$. It follows that L is a basis in $\mathbb{R}^{n-1}$ with $|\det(L)| = 2^{\ell(n-1)}$. $\{p_1, p_2, \dots, p_{n-1}\}$ is a random permutation of the set $\{1, 2, \dots, n\} \setminus \{\hat{\imath}\}$ (where $\{\hat{\imath}\}$ is defined in [1]), and B is a matrix such that

$$\begin{array}{ll} B_{p_i,j} = L_{i,j} & \text{for } i,j = 1,2,\dots,n-1 \\ B_{\hat{\imath},j} = 0 & \text{for } j = 1,2,\dots,n-1 \\ B_{i,n} = b_i(\ell) & \text{for } i = 1,2,\dots,n. \end{array}$$

It follows that B is a permutation of the rows and the columns of a lower triangular matrix whose diagonal elements are either -2^ℓ or 2^ℓ. Therefore B is a basis in $\mathbb{R}^n$ and $|\det(B)| = 2^{\ell n}$.

B' is obtained by permuting the columns of B, and therefore the columns of B' form a basis of $\mathbb{R}^n$. Furthermore, $|\det(B')| = |\det(B)| = 2^{\ell n}$.

One of the proposed versions of LTMADS uses a minimal positive basis at every iteration, and the other variant uses a maximal positive basis at every iteration. The columns of $[B' \ \ -b']$ with $b'_i = \sum_{j\in N} B'_{ij}$ define a minimal positive basis, and the columns of $[B' \ \ -B']$ define a maximal positive basis [3].

Therefore, if $D_k = [B' \ -b']$ or if $D_k = [B' \ -B']$, then D_k has all integral entries in the interval $[-n2^\ell, n2^\ell]$ or in the interval $[-2^\ell, 2^\ell]$, respectively $\left(\text{with } 2^\ell = \frac{1}{\sqrt{\Delta_k^m}} \text{ and } \ell \in \mathbb{N}\right)$. It follows that each column d of D_k can be written as a nonnegative integer combination of the columns of $D = [I \ \ -I]$. Hence, the frame defined by D_k is on the mesh M_k.

Two cases must be considered to show the second bullet. If the maximal positive basis construction is used, then $\|\Delta_k^m d\|_\infty = \Delta_k^m \|d\|_\infty = \sqrt{\Delta_k^m} = \Delta_k^p$. If the minimal positive basis construction is used, then $\|\Delta_k^m d\|_\infty = \Delta_k^m \|d\|_\infty \le n\sqrt{\Delta_k^m} = \Delta_k^p$. The proof of the second bullet follows by noticing that $\max\{\|d'\|_\infty : d' \in [I \ \ -I]\} = 1$.

To show the third bullet, one must now verify that the limit of the normalized sets $\overline{D_k} := \{\frac{d}{\|d\|_\infty} : d \in D_k\}$ forms a positive basis. Following Coope and Price [2], we first need to show that $|\det(\overline{B'_k})|$ is bounded below by a positive constant that is independent of k. This is true since $|\det(\overline{B'_k})| = 1$. Furthermore, since normalized directions are used, it follows that the limit of $\overline{B'_k}$ is a basis in $\mathbb{R}^n$. To show that the limit of $\overline{D_k}$ is a positive basis we next need to verify one of the conditions (C1) or (C2) in [2], concerning the columns added to each basis to form a positive basis. In the case of the maximal bases, condition (C1) is easily satisfied. For the minimal bases, (C2) holds since all the structure constants ξ (again following the definition of Coope and Price [2]) satisfy $-1 \le \xi \le -\frac{1}{n}$. $\square$

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